

DMA-SAP-431 APPLIED LINEAR ALGEBRA

SEMESTER: Spring
CREDITS: 6 ECTS (4 hrs. per week)
LANGUAGE: English
DEGREES: SAPIENS program

Course overview

This is a foundation course in linear algebra. By its nature, linear algebra has many applications in abstract mathematics and in real life. We will present theoretical concepts with their motivation and applications.

The class time will be devoted to lectures where the students should gain an understanding of basic concepts and methods, realize connections between various parts of linear algebra and eventually build a global picture of linear algebra. The material we cover is also meant as an introduction to a more abstract level of learning or using mathematics.

Throughout the course, we will incorporate Python programming as a way to both perform necessary computations and to explore the algorithms and techniques of Linear Algebra. No Python background is assumed, although it is assumed that all students either have some basic programming experience or are taking an introductory programming class concurrent to this course. Students will use Python to implement algorithms, solve problems, and visualize geometric interpretations. Students will learn to manipulate vectors and matrices, solve systems of linear equations, and explore matrix algebra including multiplication, inverses, determinants, and rank. Computational methods will be applied to study eigenvalues, eigenvectors, and diagonalization, with practical applications such as dynamical systems and Fibonacci sequences. Visualization tools with matplotlib are used to illustrate linear transformations such as rotations and scalings.

Prerequisites

Basic knowledge of Calculus and Algebra.

Course contents and methodology

Methodology

Lecture, solving calculation problems during exercises.

Contents

- 1. LINEAR SYSTEMS (Chapter 1 in [1]).**
 - 1.1 Systems of Linear Equations
 - 1.2 Row Reduction and Echelon Forms
 - 1.3 Vector Equations
 - 1.4 The Matrix Equation $Ax=b$
 - 1.5 Solution Sets of Linear Systems
 - 1.6 Applications of Linear Systems
 - 1.7 Linear Independence
- 2. VECTOR SPACES (Chapters 2 and 4 in [1]).**
 - 2.1 Vector Spaces and Subspaces.
 - 2.2 Subspaces of $\mathbb{R}^n$
 - 2.3 Linearly Independent Sets; Basis. Dimension and rank.
 - 2.4 Coordinate Systems
 - 2.5 The Dimension of a Vector Space
 - 2.6 Change of Basis
- 3. LINEAR TRANSFORMATIONS (Chapters 1 and 4 in [1]).**
 - 3.1 Introduction to Linear Transformations
 - 3.2 The Matrix of a Linear Transformation
 - 3.3 Null Spaces and Linear Transformations
- 4. EIGENVALUES AND EIGENVECTORS (Chapter 5 in [1]).**
 - 4.1 Eigenvectors and Eigenvalues
 - 4.2 The Characteristic Equation
 - 4.3 Diagonalization
 - 4.4 Eigenvectors and Linear Transformations
 - 4.5 Discrete Dynamical Systems
- 5. ORTHOGONALITY (Chapter 6 in [1]).**
 - 5.1 Inner Product, Length, and Orthogonality
 - 5.2 Orthogonal Sets
 - 5.3 Orthogonal Projections
 - 5.4 The Gram-Schmidt Process
- 6. QUADRATIC FORMS (Chapter 7 in [1]).**
 - 6.1 Diagonalization of Symmetric Matrices
 - 6.2 Quadratic Forms
 - 6.3 Constrained Optimization
 - 6.4 The Singular Value Decomposition

Laboratory Activities in Python

- Implementation of Gaussian elimination and LU decomposition using *NumPy*.
- Visualization of vector operations (dot product, cross product) in 2D and 3D with *matplotlib*.
- Solving systems of linear equations with *numpy.linalg.solve* and comparing computational efficiency.
- Exploring matrix algebra: multiplication, powers, inverses, determinants, and rank with *NumPy*.
- Eigenvalues and eigenvectors computation with *numpy.linalg.eig* and applications to dynamical systems.
- Diagonalization and matrix powers applied to the Fibonacci sequence.
- Orthogonality and projections: implementing the Gram-Schmidt process in Python.
- Least-squares approximation for overdetermined systems using *numpy.linalg.lstsq*.
- Interactive visualization of linear transformations (rotations, scalings, shears) in 2D and 3D using *matplotlib*.

Textbooks

- [1] *Linear Algebra and its Applications*, 4th Edition, David C. Lay (basic bibliography).
- [2] *Linear Algebra with Applications*, 4th Edition, Otto Bretscher (additional bibliography).

Grading

The overall grade will be obtained as follows:

- Three midterms (15%, 25% and 35% respectively)
- Homework (25%)

The students whose grades are less than 5 or those who want to improve their previous grades will do a final exam the last day of the course. Their final grade will be the maximum between the grade obtained in this final exam and the result of computing 70% of the final exam and 30% of the homework.

The students who fail the course will have the chance to do an extraordinary exam. The grade obtained in this exam will be their definitive grade.

The exams are all closed notebook, closed textbook and no calculator. The course will not be graded on a curve, i.e., there is no bound on the numbers A's, B's, C's, etc.

Use of AI

The use of AI to create entire works or relevant parts, without citing the source or the tool, or without explicit permission in the assignment description, will be considered plagiarism and will be regulated in accordance with the University General Regulation.